LETTERS TO THE EDITORS-IN-CHIEF

Comment on "The Nakayama-Koyama approach to laminar forced convection heat transfer to power-law fluids," by H. I. Andersson

To the Editors-in-Chief:

I read with great interest Dr. Helge I. Andersson's note[1] in which he carefully examined the accuracy and applicability of our high Prandtl number asymptotic expression for forced convection heat transfer to non-Newtonian power-law fluids.[2] I wish to thank Dr. Andersson for pointing out that the algebraic equation (5) for the momentum shape factor Λ fails to give a realistic solution, when the power-law index n is less than its critical value n_{cr} which, I realize, can be given by

$$n_{cr}=\frac{3m-2+[(3m-2)^2+315m^2]^{1/2}}{21m}$$

It is also gratifying to know that our high Prandtl number asymptote (Eq. (8)) is applicable even for the range $n<n_{cr}$, if exact values are used for the momentum shape factors.

However, I do not agree with Dr. Andersson that our high Prandtl number asymptote (8) is valid only for the cases in which

$$\mathrm{Pr}_x \propto x^{(3m-1)(n-1)/(n+1)} \to \infty \text{ as } x\to 0,$$

corresponding to the range indicated as shaded areas in Figure 4. In what follows, I will show that our high Prandtl number asymptote (8) is valid for all realistic combinations of the power-law index n and the Falkner-Skan parameter m, and hence for most practical problems associated with power-law fluids. According to the low Prandtl number asymptote (12) and the high Prandtl number asymptote (8), the exponent of the apparent Prandtl number Pr_x changes from 1/2 to 1/3, as Pr_x increases. The apparent Prandtl number Pr_{xtr} where two asymptotes intersect each other may be obtained by equating Eqs. (8) and (12), as

$$\mathrm{Pr}_{xtr}=\frac{\left(\dfrac{m}{6nC^{n-1}\Lambda}\right)^{2/(n+1)}\left\{\dfrac{2(2+\Lambda)}{15}\left(1+m-\dfrac{(n-1)(3m-1)}{3(n+1)}\right)\right\}^2}{\left\{\dfrac{3}{10}(1+m)\right\}^3}$$

Thus, the high Prandtl number asymptote can be used when

$$\mathrm{Pr}_x>\mathrm{Pr}_{xtr}$$

or equivalently, when

$$x/L>(x/L)_{tr} \quad \text{for } (n-1)(3m-1)>0$$

and

$$0<x/L<(x/L)_{tr} \quad \text{for } (n-1)(3m-1)<0$$

where

$$(x/L)_{tr}=a^{3/(1-3m)}(\mathrm{Pr}_{xtr}/\mathrm{Pr}_L)^{(n+1)/(n-1)(3m-1)}$$

is the transition point determined by substituting Eqs. (1) and (8) into the foregoing apparent Prandtl number inequality. Furthermore, we note that the characteristic Prandtl number Pr_L is usually quite large,[3] while Pr_{xtr} and a are the order of unity. Thus,

$$(x/L)_{tr}\ll 1 \quad \text{for } (n-1)(3m-1)>0$$

and

$$(x/L)_{tr}\gg 1 \quad \text{for } (n-1)(3m-1)<0$$

Therefore, the inequality $\mathrm{Pr}_x>\mathrm{Pr}_{xtr}$ always holds essentially over the whole heat transfer surface, and so does the high Prandtl number asymptote (8).

Let us estimate $(x/L)_{tr}$ for the case we treated, and discussed by Dr. Andersson in his note, namely, the stagnation flow on a circular cylinder in cross flow. From Eq. (5), we find $\Lambda=0.4911$ for $m=1$ and $n=1.6$, and evaluate

$$\mathrm{Pr}_{xtr}=0.3465$$

which gives, when $a=0.92$,

$$(x/L)_{tr}=7.77\times10^{-4} \text{ (i.e., 0.0445 deg.)} \quad \text{for } \mathrm{Pr}_L=10$$

and

$$(x/L)_{tr}=5.29\times10^{-6} \text{ (i.e., 0.0003 deg.)} \quad \text{for } \mathrm{Pr}_L=100$$

Hence, the high Prandtl number asymptote fails only for a very small stagnation spot, $0\leqslant x/L\leqslant(x/L)_{tr}$, where the boundary layer theory is not applicable anyway. Possible inaccuracy at this stagnation spot (due to the failure of the high Prandtl number assumption at the spot) never affects the overall accuracy of the heat transfer estimate based on our high Prandtl number asymptote (8), since possible upstream errors always tend to diminish downstream due to the parabolic nature of boundary layer flows. Dr. Andersson does not seem to be aware of the foregoing fact.

References

1 Andersson, H. I. The Nakayama-Koyama approach to laminar forced convection heat transfer to power-law fluids. *Int. J. Heat Fluid Flow*, 1988, **9**, 343–346

2 Nakayama, A. and Koyama, H. An asymptotic expression for forced convection in non-Newtonian power-law fluids. *Int. J. Heat Fluid Flow*, 1986, **7**, 99–101

3 Acrivos, A. M., Shah, M. J. and Petersen, E. E. Momentum and heat transfer in laminar boundary layer flows of non-Newtonian fluids past external surfaces. *AIChE J.*, 1960, **6**(2), 312–317

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Response

To the Editors-in-Chief:

Professor Nakayama's arguments in favor of the high Prandtl number asymptote (8)* is certainly correct for the dilatant

* Equation and reference numbers are as in the Technical Note.